MATH 118: Quiz 6

Name: key

Directions:

- Show your thought process (commonly called "showing your work") when solving each problem for full credit.
- If you do not know how to solve a problem, try your best and/or explain in English what you would do.
- Good luck!

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1. Suppose $P(x)$ is a polynomial and c is a real number. Write down the definition of a zero for c.

   - c is a x-intercept of the graph of P
   - $x=c$ is a solution to the equation $P(x)=0$. Meaning, $P(c)=0$.
   - $(x-c)$ is a factor of $P(x)$

2. Suppose

$$P(x) = 3x^{123} - 4x^{99} + 5x^{72} - 5x^{19} + 4x^2 - 3x^2 - 1$$

   is divided by $x-1$. What is the remainder?

By the remainder theorem, the remainder is $P(1)$.

$$P(1) = 3\cdot 1 - 4\cdot 1 + 5\cdot 1 - 5\cdot 1 + 4\cdot 1 - 3\cdot 1 - 1 = \boxed{-1}$$

3. True or False: A complete factorization of $P(x)$ over $\mathbb{R}$ always results in linear factors.

False. could have irreducibles.

4. Suppose $P(x) = x^3 + 3x^2 + 4x + 12$.

(a) How many zeros does $P(x)$ have?

Degree 3. So 3 zeros.

(b) Find a complete factorization over $\mathbb{R}$.

Grouping. $P(x) = x^3 + 3x^2 + 4x + 12$

$= x^2(x+3) + 4(x+3)$

$= (x+3)(x^2+4)$

$x^2 + 4$ irreducible because

$b^2 - 4ac = 0^2 - 4 \cdot 1 \cdot 4$

$= 0 - 16$

< 0

(c) Find a complete factorization over $\mathbb{C}$.

Now factor irreducible $x^2 + 4$.

Solve $x^2 + 4 = 0$.

$x^2 = -4$

$\sqrt{x^2} = \pm\sqrt{-4}$

$x = \pm i\sqrt{4}$

$x = \pm 2i$

So

$P(x) = (x+3)(x-2i)(x-(-2i))$

$= (x+3)(x-2i)(x+2i)$